What is a Wavefunction?

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The fact that the wavefunction of a many-body quantum mechanical system is a function on a high-dimensional configuration space, rather than our familiar 3-dimensional space or 4-dimensional spacetime, has led to the suggestion, by David Albert and others,[1] that any realist interpretation of quantum mechanics must regard this high-dimensional space as the real arena in which events take place, something more fundamental than our familiar spacetime. In this talk, I will argue that this conclusion is not warranted.

Underlying the reasoning that leads to the conclusion is an analogy between a wavefunction and a classical field. A field is an assignment of values (in this case, complex numbers) to an underlying space. If a field regarded as part of the fundamental ontology of the theory, then its substratum, the space on which it is defined, should be regarded as the fundamental space of the theory, the arena in which events transpire.

This, or something like it, is the argument. It, together with much (in fact, almost all) of the literature on wavefunction ontology, is expressed in terms of nonrelativistic quantum mechanics. We know, however, that this is not a fundamental theory; it is a low-energy, nonrelativistic approximation to a relativistic quantum field theory.

The question I want to address is: how do things look if we regard (as we should) the wavefunctions of nonrelativistic quantum mechanics, not as fundamental, but as emergent, in an appropriate (non-relativistic, low-energy) regime, from structures defined in quantum field theory? To this end, I will give a sketch of how wavefunctions arise from quantum field theory. Despite occasional remarks in the literature that nothing essential changes if we make the move to quantum field theory (the same conclusion follows, it is said, but the dimensionality of spacetime becomes infinite),[2] I will argue that the reasoning that leads to the conclusion that the physical arena is a high-dimensional spacetime loses much of its apparent force when we think of wavefunctions in this way.

The basic structures of quantum field theories are field operators associated with spacetime points, from which the observables of the theory are constructed, and quantum states, which can be represented as a Hilbert-space vector on which these operators act, or, alternatively, as a functional assigning expectation values to observables. The quantum state yields the basic ontology of the theory.[3] States of a fixed, finite particle number will be exceptional states; a general state will be a superposition of n particle states, for arbitrarily large n. We can (subject to certain qualifications, which can be specified) represent a quantum field-theoretic state by giving, for each n, an n-particle wavefunction, defined on a

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[1] See Albert (1996, 2013), Monton (2002, 2006), Lewis (2004), Ney (2012), and the papers in Ney and Albert, eds. (2013).

[2] See, *e.g.*, Ney (2013), 48.

[3] This is the most natural way of construing the ontology of quantum field theory, and is either implicit or explicit in most discussions of the ontology of quantum field theory. It is has recently been dubbed *Spacetime State Realism* by Wallace and Timpson (2010).

$3n$-dimensional configuration space. These configuration spaces are constructed from field operators defined on ordinary spacetime. An n-particle state selects out one of these $3n$-dimensional configuration spaces, and it is only in such a state that we can represent the quantum state by a wave-function on that configuration space.

When we reflect on the physical meaning of these wavefunctions, we will see that, though we can talk of the value of, say, a one-particle wavefunction at a point, this value is not a local property of the point but a global property of the quantum state. This is because, for states, such as (for example) 2 or 3 particle states, that are orthogonal to all one-particle states, one-particle wavefunctions will either be undefined or will be identically equal to zero.[4] To see that this means that the value of a one-particle wavefunction at a point is not a local beable: Consider two quantum states. The first is a one-particle state. The second is a two-particle state, which differs only from the first state by the addition of a second particle confined to a finite region R. From the first state, we can construct a one-particle wavefunction; suppose that this has nonzero values at some points far from the region R. The second state, since it is a two-particle state, yields a one-particle wavefunction that is zero everywhere. A nonzero value for a one-particle wavefunction at a point spacelike separated from R is incompatible with there being a particle located with certain in R, and hence is not a local property of that point.

How does this way of thinking about a wavefunction affect arguments about the dimensionality of spacetime?

First, the argument sketched above invokes an analogy between a wavefunction and a classical field. The assignment of a field value to a point is, crucially, a *local* matter of fact. A field value is a *local beable*, in Bell's terminology. It falls out from our consideration of how wavefunctions emerge from quantum field theory that a wavefunction's assignment of a value to a spacetime point is *not* a local matter of fact. This is true even in the case of a single-particle wavefunction, the case most amenable to the analogy between the wavefunction and a field.

Second, the basic structures of the theory are, as we have already mentioned, operators defined on ordinary 4D spacetime, together with the quantum state. Wavefunctions and the configuration spaces on which they are defined are derivative structures, defined in terms of the quantum state and operator-valued fields on ordinary 4D spacetime. Though wavefunctions can be regarded as real, they are not fundamental, and we should be cautious about basing conclusions about fundamental ontology on them.

Thus, the conception of wavefunctions on which the conclusion about the dimensionality of spacetime rests, namely, that they are fields that live on a high-dimension space that is more fundamental than ordinary spacetime, fails on two counts. Wavefunctions are not fields at all, and the configuration space on which they are defined is constructed, via procedures valid only in a low-energy, nonrelativistic approximation.

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[4] This is not meant to be intuitively obvious, but it is a consequence of the explicit construction of one-particle wavefunctions, which will be sketched in the talk.

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